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Black hole thermodynamics with the cosmological constant as independent variable: Bridge between the enthalpy and the Euclidean path integral approaches

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ABSTRACT

Viewing the cosmological constant $\Lambda < 0$ as an independent variable, we consider the thermodynamics of the Schwarzschild black hole in an anti-de Sitter (AdS) background. For this system, there is one approach which regards the enthalpy as the master thermodynamic variable and makes sense if one considers the vacuum pressure due to the cosmological constant acting in the volume inside the horizon and the outer size of the system is not restricted. From this approach a first law of thermodynamics emerges naturally. There is yet another approach based on the Euclidean action principle and its path integral that puts the black hole inside a cavity, defines a quasilocal energy at the cavity's boundary, and from which a first law of thermodynamics in a different version also emerges naturally. The first approach has affinities with critical phenomena in condensed matter physics and the second approach is an ingredient necessary for the construction of quantum gravity. The bridge between the two approaches is carried out rigorously, putting thus the enthalpic thermodynamics with Λ as independent variable on the same footing as the quasilocal energy approach.

1. Introduction

In recent years, a new direction in gravitational and black hole thermodynamics appeared. It is based on treatment of the cosmological constant $\Lambda < 0$ as a thermodynamic variable and leads to a number of interesting physical consequences in which the thermodynamic potential enthalpy emerges naturally and a close analogy with van der Waals forces and critical phenomena in condensed matter physics can be carried out. The works and reviews on the subject can be found in [1–7]. These nontrivial features arise in cases like the Schwarzschild and Reissner–Nordström black holes in an anti-de Sitter (AdS) background [4]. A main feature is the first law of thermodynamics in which the term $d\Lambda$ is taken into account with Λ acting as a vacuum pressure P. Indeed P takes the value $P = -\frac{\Lambda}{8\pi}$ and is conjugate to a thermodynamic volume V given by $V = \frac{4}{3}\pi r_+^3$, r_+ being the horizon radius. Substantiation of this approach is based on precise derivations as well as heuristic arguments that take into account the volume inaccessible to an observer due to the existence of a horizon, whereas the outer size of the black hole thermodynamic system is not restricted in this approach.

On the other hand, there exists a well-defined gravitational black hole thermodynamics based on the Euclidean action principle and path integral approach which is recognized as an ingredient necessary for the construction of a quantum gravity [8–10]. This approach was put on a firm basis taking into account the finiteness of the system as an essential ingredient, i.e., the black hole is constrained to lie inside a cavity. At the cavity's boundary a temperature T is prescribed, defining thus a canonical ensemble [11–15], or if electric charge is present a grand canonical ensemble should be used [16,17]. Also at the cavity's boundary one can define a quasilocal energy and from it extract the first law of thermodynamics. In this approach the horizon turns into a bolt [9] and the volume under the horizon becomes totally irrelevant, whereas the outer size of the black hole thermodynamic system is required to have a well defined boundary.

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